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Abstract: We study quenched SU(2) lattice gauge theory with adjoint fermions in a wide range of temperatures. We focus on spectral quantities of the Dirac operator and use the temporal fermionic boundary conditions as a tool to probe the system. We determine the deconfinement temperature through the Polyakov loop, and the chiral symmetry restoration temperature for adjoint fermions through the gap in the Dirac spectrum. This chiral transition temperature is about four times larger than the deconfinement temperature. In between the two transitions we find that the system is characterized by a non-vanishing chiral condensate which differs for periodic and anti-periodic fermion boundary conditions. Only for the latter (physical) boundary conditions, the condensate vanishes at the chiral transition. The behavior between the two transitions suggests that deconfinement manifests itself as the onset of a dependence of spectral quantities of the Dirac operator on boundary conditions. This picture is supported further by our results for the dual chiral condensate.

Contents

1 Introductory remarks

Confinement and chiral symmetry breaking are two outstanding properties of Quantum Chromodynamics (QCD), shaping all of nuclear physics. The emergence of both these phenomena depends on non-perturbative mechanisms, but it is an open question if and how the respective mechanisms are related, or what they are in detail. Each phenomenon is connected with a particular symmetry becoming broken or restored in certain limits of the theory.

From the moment on, at which the existence of phase transitions in QCD was realized, the question whether deconfinement and chiral symmetry restoration are related to different transition temperatures, $T_{\rm dec}$ and $T_{\rm ch}$, was posed. Because of the dual role of quarks, with quarks being confined on one hand, and their role in the (chiral) hadron dynamics on the other hand, this question was asked in the first instance about fermions in the fundamental representation. For this case a consensus [1] based on lattice gauge theory calculations has formed that, at least as long as no finite baryonic chemical potential μ is involved, the temperature driven phase transition happens at roughly the same temperature, $T_{\rm dec} \simeq T_{\rm ch}^{(f)}$, where we use the superscript (f) to indicate the fundamental representation.[1]

Fermions, and thus implicitly also chiral symmetry, play a role also outside QCD, e.g., for model building beyond the standard model [2]. In particular in many of those theories [3], like supersymmetry and technicolor, fermions in other representations appear, in particular adjoint ones. There is no *a priori* reason to expect the same transition temperature for such fermion representations.

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[1]For most quark masses, including likely the physical ones, there is a crossover instead of a genuine phase transition [1] in full QCD. Thus, there is no qualitative distinction of the low- and high-temperature phase, despite their historic names. In quenched QCD ("gluodynamics"), however, a second or first order phase transition exists.

Such gauge theories with adjoint fermions have been investigated since the early days of lattice simulations [4–9]. In this case, it is well established that, at least for the gauge groups investigated so far, the deconfinement temperature T_{dec} and the chiral restoration temperature $T_{\mathrm{ch}}^{(a)}$ of adjoint quarks do not coincide, the latter being generally significantly larger than the former.

Aside from the practical considerations of theories beyond the standard model, this requires that any mechanism proposed as an explanation for the equality $T_{\mathrm{dec}} \simeq T_{\mathrm{ch}}^{(f)}$ must at the same time provide an explanation for the inequality $T_{\mathrm{dec}} \neq T_{\mathrm{ch}}^{(a)}$. Such attempts have been made already. E. g., to capture the characteristic temperatures $T_{\mathrm{ch}}^{(r)}$ of chiral symmetry restoration for some representation r, a hypothetical Casimir scaling law has been proposed [10, 11], $C_2^{(r)} g^2(T_{\mathrm{ch}}^{(r)}) = \mathrm{const} \approx 4$, and discussed in the light of early lattice results in refs. [4, 5]. Here $C_2^{(r)}$ is the eigenvalue of the quadratic Casimir operator that characterizes the fermion representation r, and g^2 is the running coupling.

Out of the necessity to explain the difference it is also possible to construct a virtue: The fact that for adjoint fermions the temperatures for deconfinement and chiral symmetry restoration are different, makes such theories an important testbed to study confinement and chiral symmetry breaking individually. In this way one may hope to understand mechanisms responsible for the two phenomena and to identify possible aspects shared by both.

This point of view is the motivation for the present work: In a series of recent papers [12–16] the question of a possible connection between confinement and chiral symmetry breaking has been attacked by constructing new combined observables which are sensitive to both, confinement and chiral symmetry breaking. One example is the "dual chiral condensate" [13], which is obtained as the first Fourier component of the chiral quark condensate with respect to a generalized temporal boundary condition for the fermions. It may be shown [13] that the dual chiral condensate for fundamental fermions is a sum of generalized (i.e., non-straight) Polyakov loops and thus is an order parameter for center symmetry and therefore for confinement (at least in the quenched case). On the other hand, since it is built from the usual chiral condensate, it is also sensitive to chiral symmetry breaking.

This observable can be used to characterize the deconfinement temperature T_{dec} as the one above which spectral quantities of the Dirac operator (e.g., the chiral condensate) become sensitive to a change of the temporal fermion boundary conditions [13]. The origin of the tie to chiral symmetry is the density of Dirac eigenvalues at the origin, due to the Banks-Casher relation [17]. This characterization is further underlined by the fact that both the chiral condensate and the gap in the Dirac spectrum above T_{dec} are quantities that depend on the fermionic boundary conditions (relative to the phase of the Polyakov loop) [18–20]. For the case of SU(2) Yang-Mills theory the detailed circumstances suggest the following microscopic explanation [21] for that dependence: For periodic boundary conditions low-lying modes exist that are localized on "light dyons", whereas "heavy dyons" are suppressed. The latter would otherwise be carriers of low-lying modes under anti-periodic boundary conditions. In this picture, the different abundance of light and heavy dyons in turn results from the non-vanishing fundamental Polyakov loop.

The central motivation for the present investigation is the question to what extent

the interrelations between deconfinement, chiral symmetry restoration and the fermionic boundary conditions carry over to the case of fermions in the adjoint representation of the gauge group. Of particular interest is the behavior in the intermediate phase, i.e., at temperatures $T_{\text{dec}}(\simeq T_{\text{ch}}^{(f)}) \leq T \leq T_{\text{ch}}^{(a)}$. For this range we will show that the condensate is still finite and no spectral gap has opened, but the fermionic quantities do already feel the boundary conditions. Concerning the dual chiral condensate we will establish that it is sensitive to both the deconfinement and chiral restoration transitions.

2 Setup of the calculation

In our analysis we study quenched SU(2) configurations generated with the Symanzik improved gauge action [22] using the fundamental representation. We explore a wide range of inverse couplings β, between $\beta = 2.5$ and $\beta = 4.6$, increasing β in steps of $\Delta\beta = 0.1$. Using Metropolis updates, for each value of β we generate 100 configurations in each of our ensembles on two volumes, $N^3 \times N_T = 10^3 \times 4$ and $12^3 \times 4$.

The fundamental gauge links $U_\mu(x)$ are converted to the adjoint representation

$$U_\mu^{adj}(x)_{ab} \equiv \frac{1}{2} \text{Tr}\,[\,\sigma^a\, U_\mu(x)^\dagger\, \sigma^b\, U_\mu(x)\,]\,, \qquad (2.1)$$

where σ^a, $a = 1, 2, 3$ are the Pauli matrices. The adjoint links are used in the massless staggered lattice Dirac operator (we set the lattice spacing to $a = 1$)

$$D(x,y) = \sum_\mu \eta_\mu(x)\,[\,U_\mu^{adj}(x)\,\delta_{x+\hat{\mu},y} - U_\mu^{adj}(x - \hat{\mu})^\dagger\,\delta_{x-\hat{\mu},y}\,]\,, \qquad (2.2)$$

where $\eta_\mu(x)$ is the staggered sign function $\eta_\mu(x) = \prod_{\nu=1}^{\mu-1}(-1)^{x_\nu}$.

For the staggered lattice Dirac operator we evaluate complete eigenvalue spectra using a parallel implementation of standard linear algebra routines. The staggered Dirac operator is anti-hermitean and consequently the eigenvalues λ_j are purely imaginary. The eigenvalues for the Dirac operator with mass m are then given by $\lambda_j + m$.

In our analysis we systematically explore the role of the temporal fermionic boundary conditions, which may be written as

$$\psi(\vec{x}, N_T) = e^{i\varphi}\,\psi(\vec{x}, 0)\,, \qquad (2.3)$$

where the "boundary angle" φ parameterizes the boundary condition. A value of $\varphi = \pi$ corresponds to the usual anti-periodic boundary conditions. However, here in addition we explore also periodic and more general boundary conditions, and the boundary angle φ is considered as an additional parameter to probe the system. Furthermore, for the construction of the aforementioned dual chiral condensate we need a Fourier integral over φ which is approximated by using altogether 8 values of φ in the interval $[0, 2\pi)$. To be specific, we compute complete Dirac spectra for the two boundary conditions $\varphi = 0$ and $\varphi = \pi$ for all 100 configurations in our ensembles, while spectra for the additional values $\varphi = \pi/4,\ \pi/2,\ 3\pi/4,\ \ldots$ needed for the dual chiral condensate were evaluated for subensembles consisting of only 20 configurations for each volume and β. For completeness we remark, that all other boundary conditions, i.e., the spatial fermionic boundary conditions and the boundary conditions for the gauge fields, were kept periodic.
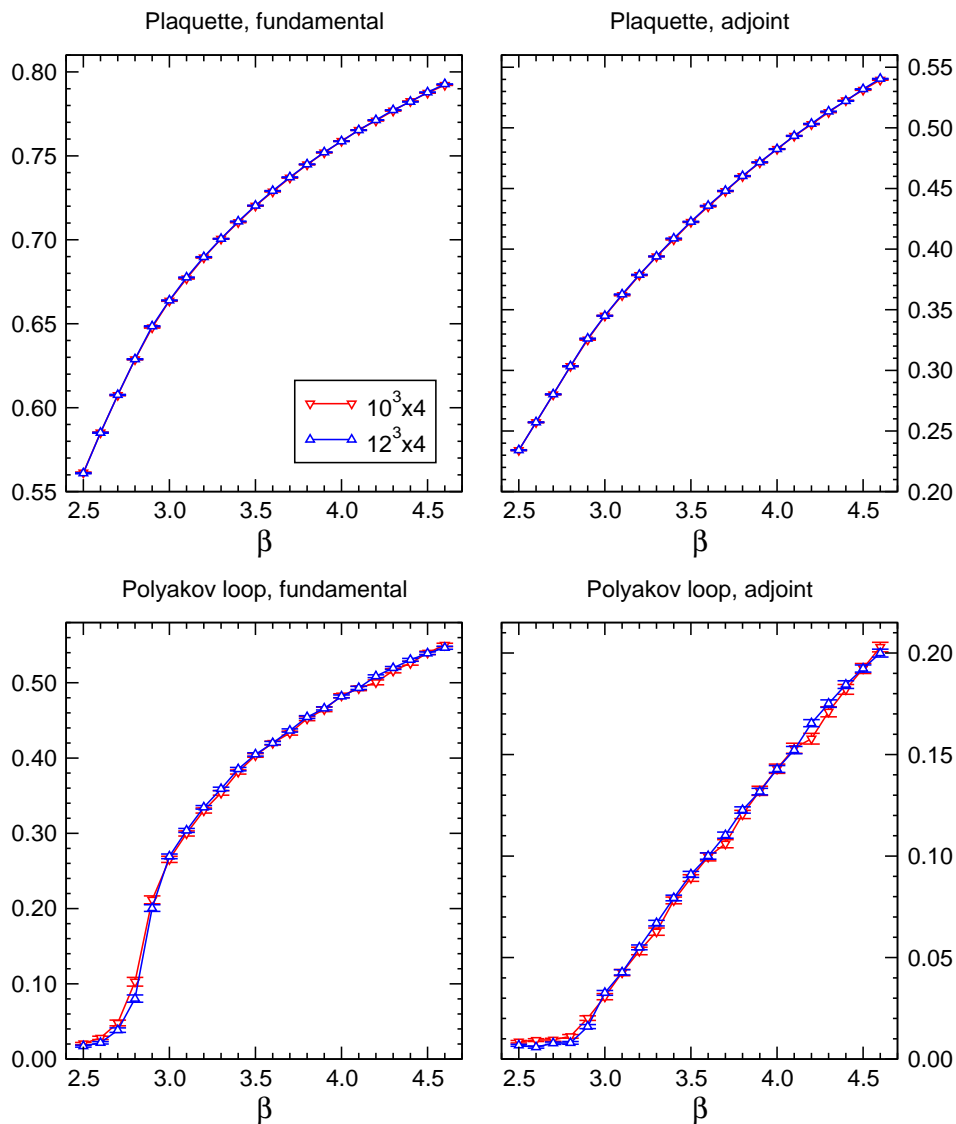
Figure 1. Gluonic observables for our quenched gauge ensembles. We show the fundamental and adjoint plaquette expectation values as a function of the inverse coupling β (top row of plots), and the fundamental and adjoint Polyakov loops (bottom row). Results for both volumes, $10^3 \times 4$ and $12^3 \times 4$ are displayed.

3 Plaquette and Polyakov loops

We begin our discussion of the numerical results with purely gluonic quantities, the plaquette expectation values and the (spatially averaged) Polyakov loops in both the fundamental and the adjoint representations.

In figure 1 we compare the fundamental and adjoint plaquette expectation values (top row) and the fundamental and adjoint Polyakov loops (bottom row) plotted as a function of the inverse gauge coupling β. The fundamental Polyakov loop can be used to determine the critical inverse gauge coupling where we observe the deconfinement transition on our

lattice with $N_T = 4$, at a value of $\beta_{\text{dec}} = 2.8$. This corresponds[2] to about $T = 215\,\text{MeV}$, and thus is very far from the continuum and infinite-volume limit of about $300\,\text{MeV}$ [24]. Our plots clearly indicate that a comparison of the data for the two different volumes, $10^3 \times 4$ and $12^3 \times 4$, reveals only very small finite volume effects.

It is a remarkable fact that also the adjoint Polyakov loop shows a changing behavior at the onset of the deconfinement transition. Since it is invariant under center transformations, there is no a-priori reason for this. This behavior will be one contribution to the sensitivity of the adjoint dual chiral condensate discussed below. However, the impact on the adjoint Polyakov loop by the breaking of center symmetry could be spurious: Adjoint fermions can be screened by a single gluon. Thus, even for static adjoint quarks string breaking occurs for all temperatures [25], and there is no deconfinement in the same sense as there is none for full QCD. Hence, in the infinite-volume and continuum limits, the adjoint Polyakov loop is non-zero in all phases. It is not an order parameter for center symmetry.

The fact that on a finite lattice an imprint of the deconfinement transition still exists has also been observed in G_2 Yang-Mills theory [26, 27], and is thus not surprising. Still, this demands caution in the interpretation of adjoint quantities, and the value of the adjoint Polyakov loop has to be interpreted rather as a lattice artifact than as a signal, as long as it cannot be unambiguously established that other effects drive its modification.

4   Spectral gap and chiral condensate

Let us now come to fermionic observables related to chiral symmetry breaking and its restoration. In this respect an important result is the Banks-Casher formula [17] which relates the chiral condensate to the density ρ of Dirac eigenvalues at the origin,

$$\langle \overline{\psi}\,\psi \rangle \;=\; -\,\pi\,\rho(0)\;. \tag{4.1}$$

This result is independent of the gauge group and its representation. As long as chiral symmetry is broken we thus expect that the eigenvalues of the Dirac operator extend all the way to the origin and build up a non-vanishing density $\rho(0)$ there. As one crosses the critical temperature, the chiral condensate vanishes and so must $\rho(0)$. At least on finite spatial volumes one observes the opening of a gap in the spectrum at a corresponding critical coupling $\beta_{\text{ch}}^{(a)}$.[3]

For the case of adjoint SU(2) we expect that chiral symmetry is restored at a higher temperature than the one where we observe deconfinement and thus expect that the spectral gap remains closed beyond $\beta_{\text{dec}} = 2.8$. This is exactly what we observe in the l.h.s. plot of figure 2. Using lattice units we show the spectral gap defined as the expectation value $\langle a|\lambda_{\text{min}}|\rangle$ of the smallest eigenvalue as a function of β. The plot clearly shows that the gap remains closed above $\beta_{\text{dec}} = 2.8$ all the way up to $\beta_{\text{ch}}^{(a)} = 3.6$, corresponding to about $870\,\text{MeV}$, where it starts to open (again we use the superscript (a) to denote the critical β for the adjoint representation). We observe that the discrepancy between the two volumes

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[2]For a string tension of $\sqrt{\sigma} = 440\,\text{MeV}$, using $a^2\sigma$ results from [23].

[3]With the currently available results it cannot be excluded that in the thermodynamic limit the gap closes such that the eigenvalues extend all the way to the origin, but still have a vanishing density $\rho(0)$ [28].
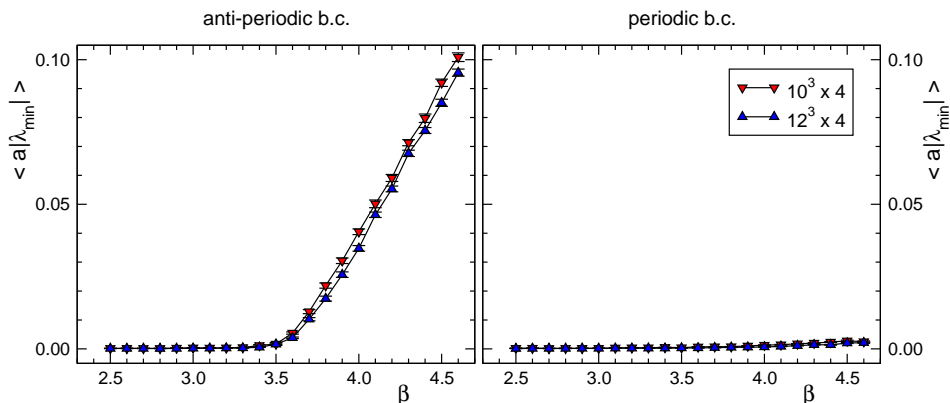
Figure 2. Spectral gap in lattice units of the Dirac operator for adjoint fermions as a function of the inverse coupling β. We compare anti-periodic temporal boundary conditions for the fermions (l.h.s. plot) to periodic boundary conditions (r.h.s.). Results for both volumes, $10^3 \times 4$ and $12^3 \times 4$, are displayed.

which are accessible to us, $10^3 \times 4$ and $12^3 \times 4$, is small with a light trend towards a smaller gap for the larger spatial volume.

One may compare the two critical inverse couplings $\beta_{\text{dec}} = 2.8$ and $\beta_{\text{ch}}^{(a)} = 3.6$ also in terms of temperatures. One finds that the deconfinement temperature and the temperature for chiral symmetry restoration behave as

$$T_{\text{ch}}^{(a)} \; \simeq \; 4(1) \, T_{\text{dec}} \; . \tag{4.2}$$

The error is a rough estimate based on the discrepancy of the determined deconfinement temperature and the known infinite-volume continuum value. Still, both transitions are different. However, they are much closer than in the case of (dynamical) SU(3) QCD, where they differ by a factor of 7.8(2) [8, 9].

Let us now discuss the r.h.s. plot of figure 2 which differs from the l.h.s. by the use of periodic temporal boundary conditions for the fermions instead of the canonical anti-periodic choice. Obviously the spectral gap remains closed when the periodic temporal boundary conditions are used for the fermions. This behavior is in agreement with what was found also for the gauge groups SU(3), SU(2) and G$_2$ in the fundamental representation [19, 20, 26].

The next step is to explore the dependence of the chiral condensate on the temperature and the boundary conditions directly. For this purpose, the condensate is determined in the same way as in [26], using both methods. In figure 3 we plot the chiral condensate in lattice units as a function of the inverse gauge coupling β and compare periodic (upper two curves) and anti-periodic (lower two curves) temporal boundary conditions for the fermions. Triangles are used for the larger $12^3 \times 4$ lattice, while the smaller $10^3 \times 4$ lattice is represented by upside-down triangles. For the anti-periodic boundary conditions we observe that the condensate remains finite up to about $\beta_{\text{ch}}^{(a)} = 3.6$, the critical value where we observed the opening of the spectral gap, and vanishes for larger β. The situation is different for the condensate with periodic boundary conditions where we find that the
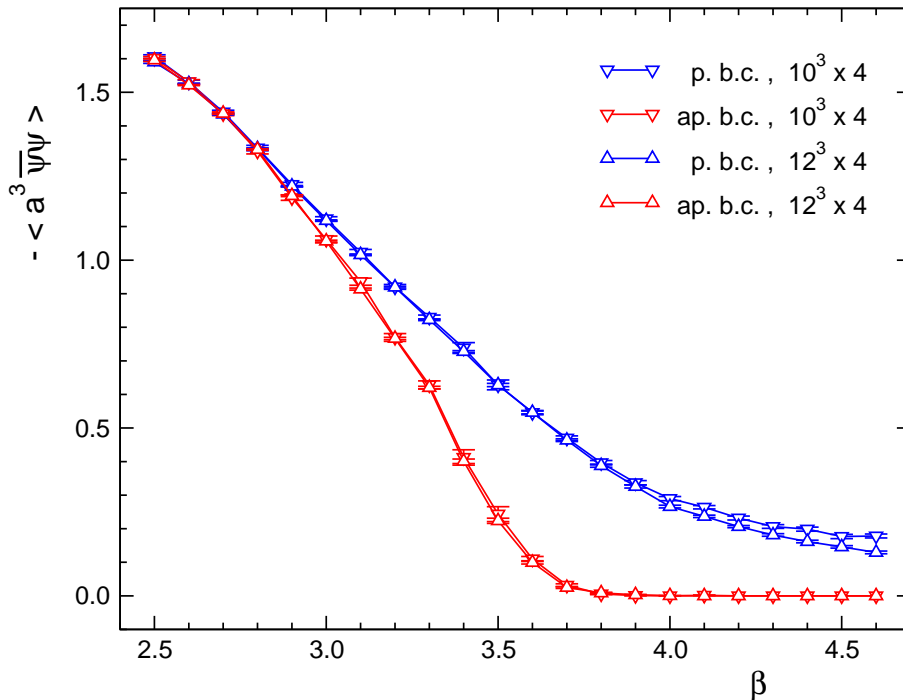
Figure 3. The chiral condensate in lattice units as a function of the inverse gauge coupling β. We show the results for both volumes, $10^3 \times 4$ and $12^3 \times 4$, and compare periodic and anti-periodic temporal boundary conditions for the fermions. Note that this is the unrenormalized chiral condensate.

condensate remains finite above $\beta_{\rm ch}^{(a)} = 3.6$, as could be already expected from the fact that no spectral gap appears (compare figure 2). Again we find that the results for the two volumes essentially fall on top of each other — only for the periodic case at the largest values of β we observe sizable finite volume effects. However, at such large values of β the spatial volume becomes so small that the results can be taken only as indicative. Still, since the major effects investigated, i. e., the chiral and deconfinement phase transitions, occur at $\beta < 4$, where no such effects are visible, the conclusions are likely not affected qualitatively by this limitation. It would be necessary to use significantly larger volumes to obtain a better systematic accuracy.

Let us finally stress an important aspect of our results for the chiral condensate: By comparing the data for periodic and anti-periodic boundary conditions at the smallest values of β we find that the results for the condensate fall on top of each other. This is true up to the value of $\beta = 2.8 = \beta_{\rm dec}$. Beyond the value of the deconfinement transition we observe that the results for the condensate at periodic and anti-periodic boundary conditions start to differ. The condensate for the anti-periodic case begins to drop relative to the periodic data until it reaches zero near $\beta_{\rm ch}^{(a)} = 3.6$. In between the two transitions we observe a finite chiral condensate for both boundary conditions but the values differ. Note that the effect is significantly stronger than the systematic finite-volume errors.

This finding underlines the characterization of deconfinement and chiral symmetry

restoration given in [13]: The deconfinement transition is characterized by the onset of a dependence of the Dirac spectrum on the fermionic boundary conditions. Chiral symmetry restoration is seen only for the physical anti-periodic boundary conditions and, according to the Banks-Casher formula, is manifest through a vanishing spectral density at the origin.

5 The dual chiral condensate

In a series of papers [12, 13, 15] observables were developed that are sensitive to both, chiral symmetry and confinement. One such observable is the dual chiral condensate Σ_1 which is defined as the first Fourier component of the chiral condensate with respect to the fermionic temporal boundary condition [13],

$$\Sigma_1 \; = \; -\frac{1}{2\pi}\int_0^{2\pi}d\varphi\, e^{-i\varphi}\left\langle \overline{\psi}\psi \right\rangle_m^{(\varphi)} \; = \; \frac{1}{2\pi V}\int_0^{2\pi}d\varphi\sum_j \frac{e^{-i\varphi}}{\lambda_j^{(\varphi)} + m}\;. \tag{5.1}$$

In our notation the superscript (φ) indicates which fermionic boundary condition is used. In the second step of (5.1) we have inserted the spectral sum for the Dirac operator. The integral over the boundary angle is approximated with 8 values of φ in the interval $[0, 2\pi)$ using the Simpson rule. This procedure was shown to give rise to uncertainties for the numerical integral in the one percent range [13], in case of the smooth dependence on the boundary angle observed here.

The dual chiral condensate Σ_1 may be viewed as a collection of generalized Polyakov loops: Like any other gauge invariant quantity on the lattice the scalar expectation value $\langle\overline{\psi}\psi\rangle_m$ can be expressed as a collection of closed loops on the lattice which are dressed with link variables. These loops may be distinguished by their winding number around the compactified time direction and the Fourier transformation with respect to the boundary angle φ in (5.1) projects to the equivalence class of loops that wind once. Consequently these loops that build up Σ_1 transform under center transformations like the conventional straight Polyakov loop. Thus in the quenched theory with (current) fermions in the fundamental representation Σ_1 serves as order parameter for center symmetry and thus for confinement. In addition, for small enough quark mass m the observable becomes sensitive to chiral symmetry breaking since it is derived from the conventional chiral condensate. In the limit of large quark mass longer loops are suppressed and Σ_1 approaches the conventional straight Polyakov loop (with a different overall normalization).

Since for adjoint fermions deconfinement and chiral symmetry restoration appear at different temperatures, it is an interesting question how the adjoint dual chiral condensate behaves in this situation. Furthermore, as discussed above, the adjoint Polyakov loops from which the adjoint dual chiral condensate is constructed, are invariant under center transformations. The behavior of the adjoint dual chiral condensate is therefore much harder to predict on general grounds than in the fundamental case.

In figure 4 we show our results for the dual chiral condensate as a function of the inverse gauge coupling β. In the l.h.s. plot we consider a situation which corresponds essentially to the chiral limit ($am = 0.005$), while on the r.h.s. we use a rather large quark mass ($am = 0.4$) where Σ_1 is expected to behave similar to the conventional straight
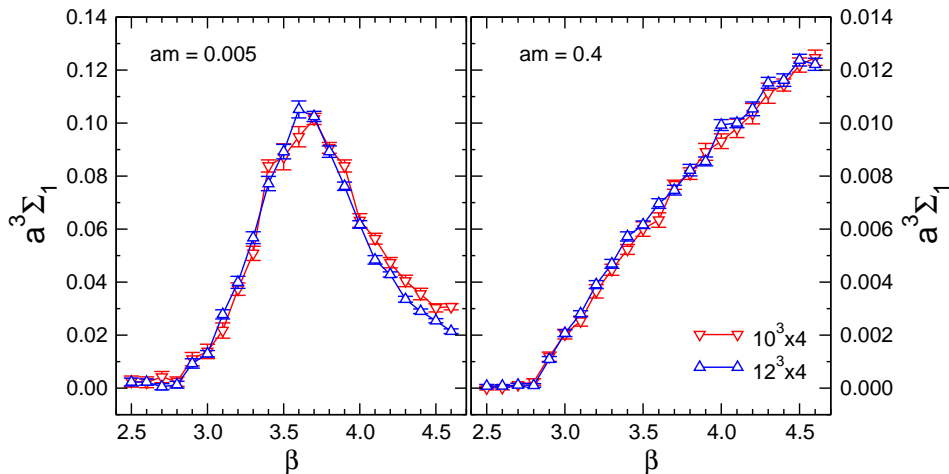
Figure 4. The dual chiral condensate Σ_1 in lattice units as a function of the inverse gauge coupling β. We show the results for both volumes, $10^3 \times 4$ and $12^3 \times 4$, and compare two different quark masses.

Polyakov loop. The plots show clearly that Σ_1 starts to rise at the deconfinement transition at $\beta_{\mathrm{dec}} = 2.8$. Since the dual chiral condensate is the first Fourier component of the condensate with respect to the fermionic boundary conditions, its behavior supports the characterization of the deconfinement transition as the onset of dependence of fermionic quantities on the femrionic boundary conditions.

Comparison of the two plots in figure 4 shows that only for the small quark mass Σ_1 also the chiral symmetry restoration at $\beta_{\mathrm{ch}}^{(a)} = 3.6$ is resolvable. In the l.h.s. plot we observe a maximum of Σ_1 at this coupling. Beyond this value we find a decreasing behavior. In the r.h.s. plot, where the rather large mass $am = 0.4$ was used, we find no signal at $\beta_{\mathrm{ch}}^{(a)} = 3.6$ and, as expected, Σ_1 behaves similar to the conventional straight Polyakov loop in the adjoint representation (compare figure 1), i.e., displays a monotonically rising behavior beyond $\beta_{\mathrm{dec}} = 2.8$.

6 Summary and discussion

In this paper we have revisited the phenomenon of different deconfinement and chiral symmetry restoration temperatures of gauge theories coupled to adjoint fermions. In our study of quenched SU(2) gauge configurations we have focused on analyzing a set of observables related to the spectrum of the lattice Dirac operator. An important tool in this analysis was the use of generalized temporal fermionic boundary conditions. The corresponding boundary angle serves as an additional parameter to probe the system.

We confirm that for adjoint fermions the deconfinement transition (determined by the Polyakov loop expectation value) and the chiral symmetry restoration (identified by the opening of a gap in the Dirac spectrum), are different with $T_{\mathrm{ch}}^{(a)} = 4(1) \, T_{\mathrm{dec}}$. We find that also fermionic quantities are affected by the deconfinement transition, in particular at the deconfinement temperature a dependence of the chiral condensate on the fermionic

boundary condition becomes manifest. Between the two transitions the system is characterized by a chiral condensate which differs for different boundary conditions, but for the physical anti-periodic boundary conditions still has not reached zero. This happens at the second transition $T_{\mathrm{ch}}^{(a)}$ where chiral symmetry is restored, i.e., the chiral condensate finally vanishes. However, we do not find any indications of this being related to a thermodynamic phase transition of the pure gauge system, at least in any gluonic observable we have investigated. This would be in line with observations for the dynamical case [8, 9]. The only affected quantities are fermionic ones: For periodic temporal fermion boundary conditions the condensate remains finite for all temperatures (i.e., all gauge couplings) we considered, though the systematic uncertainty increases quickly for $\beta > 4$. Finally we find that the dual chiral condensate indeed sees both transitions, thus providing support that this observable is sensitive to both confinement and chiral symmetry breaking. Still, a careful study of the thermodynamic limit is mandatory for a firm conclusion, in particular at temperatures beyond the chiral phase transition.

An interesting question is how the picture changes when dynamical adjoint quarks would be used. From dynamical SU(3) QCD studies [8, 9] it is known that the back-reaction of the fermions on the gluon field is minor for large masses in the sense that the chiral phase transition at $T_{\mathrm{ch}}^{(a)} \approx 7.8(2) \, T_{\mathrm{dec}}$ has no strong effect on the remaining observables. On the other side it is known that the deconfining phase transition is mainly unaffected by the presence of dynamical adjoint fermions, the chiral symmetry of which is broken for all $T < T_{\mathrm{ch}}^{(a)}$. However, in contrast to the pure SU(3) Yang-Mills theory, the deconfining phase transition is of strong first order. Nonetheless, we expect that the picture developed here, i.e., an intermediate phase where fermionic quantities do already depend on the temporal fermionic boundary conditions but the chiral condensate is still non-vanishing, carries over to the full dynamical theory, since this effect should not be affected by the order of the phase transition.

Our analysis has increased the amount of known phenomenological facts about systems with deconfinement and chiral symmetry restoration transitions. In particular the role of the fermionic boundary conditions, which have become an important issue in recent years, was clarified for a system with adjoint fermions. It is obvious that any future microscopic explanation of the deconfinement and chiral symmetry restoration transitions will have to describe the dependence on boundary conditions correctly.

A particular highlight of the results is that, since the calculations have been quenched, all the mechanisms usually associated with chiral restoration, like modification of topological properties, cannot be responsible for the restoration of the adjoint chiral symmetry: All of these effects occur at T_{dec}, unmodified in the quenched calculation. The dynamical origin of adjoint dynamics is therefore fundamentally and qualitatively different from the one for fundamental dynamics. Especially, as all dynamics driving fundamental chiral symmetry breaking cease at a quarter of the temperature $T_{\mathrm{ch}}^{(a)}$, they alone cannot provide adjoint chiral symmetry breaking. In particular, this implies a very strong adjoint-quark-gluon dynamics in the high-temperature phase to keep chiral symmetry broken, but since everything is quenched, this implies strong gluon dynamics even at $4 \, T_{\mathrm{dec}}$ (or nearly $8 \, T_{\mathrm{dec}}$ for SU(3)).

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